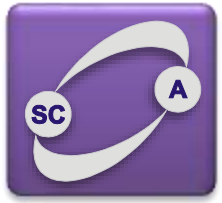


UNIVERZITET U NOVOM SADU
FAKULTET TEHNIČKIH NAUKA
KATEDRA ZA AUTOMATIKU I UPRAVLJANJE SISTEMIMA

Translatorni mehanički sistemi

Modeli fizičkih sistema

Modeliranje i simulacija sistema

Upravljanje, modelovanje i simulacija sistema

Promenljive

Osnovne promenljive:

- x – rastojanje [m]
- v – brzina [m/s]
- a – ubrzanje [m/s^2]
- f – sila [N]

Sve su funkcije vremena

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

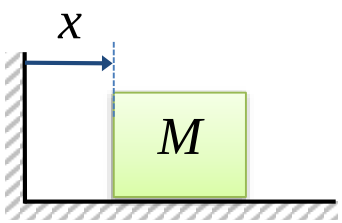
Dodatne promenljive:

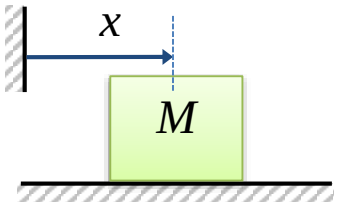
- w – energija [J]
- p – snaga [W]

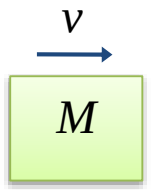
$$p = f \cdot v$$

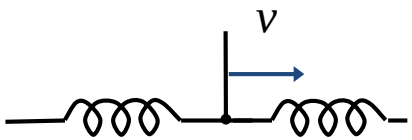
$$p = \frac{dw}{dt}$$

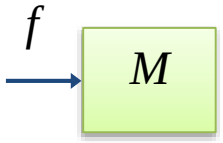
$$w(t) = \int_{t_o}^{t_1} p(t)dt + w(t_0)$$

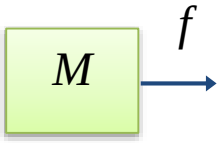












Elementi i njihovi zakoni

Posmatramo elemente i pojave:

- Masa
- Trenje
- Elastičnost

Masa tela

- Masa tela *M* [kg]
- II Njutnov zakon:

$$\frac{d}{dt}(M \cdot v) = f \qquad \text{za } M\text{=const} \qquad M\frac{dv}{dt} = f$$

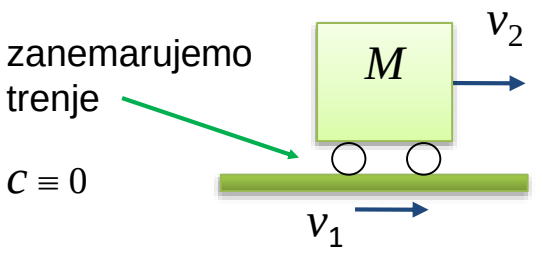
- Energija
  - Kinetička $w_k = \frac{1}{2} M \cdot v^2$
  - Potencijalna $w_p = Mgh$

Trenje

- Sila trenja se javlja kada se dva tela dodiruju i kreću različitim brzinama

$$f = f(\Delta v) \qquad \Delta v = v_2 - v_1$$

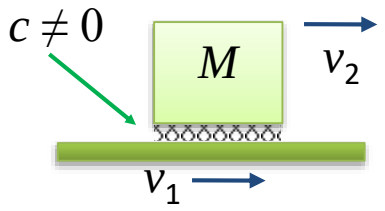
česta oznaka: c



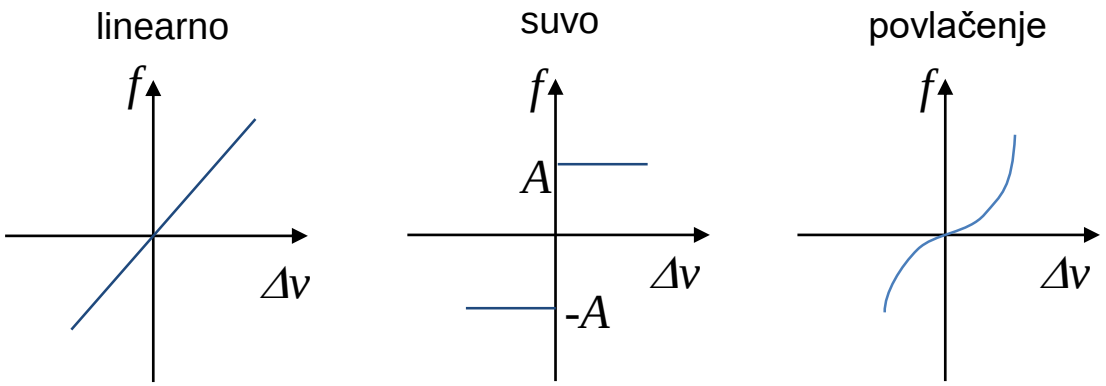
- Linearizovana zavisnost: $f = c \cdot \Delta v$

  c – koeficijent trenja (viskoznosti) [Ns/m]

  - direktno je srazmeren površi dodira, a obrnuto srazmeren debljini uljanog filma.

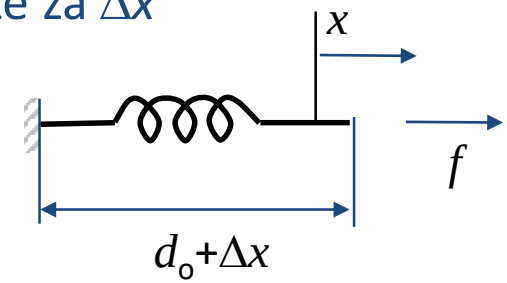


- Karakteristika trenja



Elastičnost

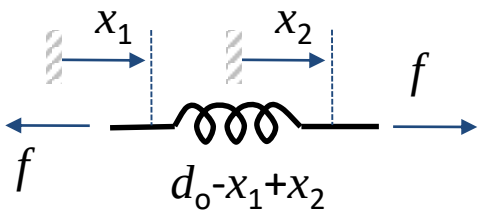
- Opruga
  - Pod dejstvom spoljašnje sile *f* opruga se isteže za Δx
    - d_o - istegnutost opruge bez dejstva sile



- Sila u opruzi: $f = f(\Delta x) \qquad \Delta x = x_2 - x_1$

- Za mala istezanja važi (linearizovano ponašanje) $f = k \cdot \Delta x$

  k - koeficijent elastičnosti [N/m]



- Energija opruge: $w_p = \frac{1}{2} k(\Delta x)^2$

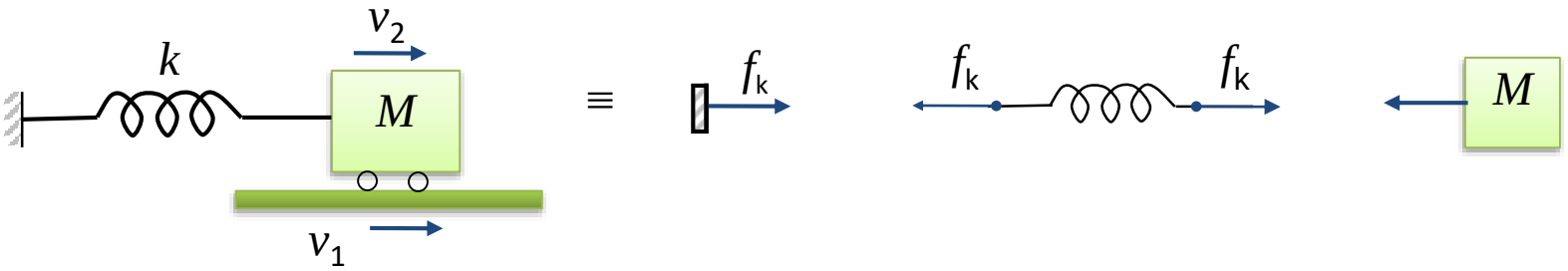
Zakonitosti kod uzajamnog dejstva elemenata

1. Dalamberov zakon (drugačuja formulacija II Njutnovog zakona)

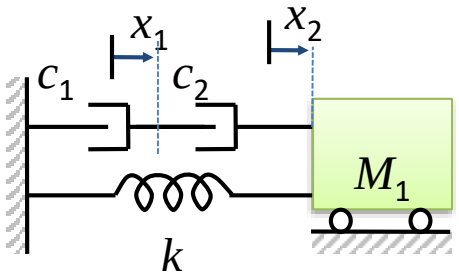
$$\sum_i (f_{ext})_i = M\frac{dv}{dt} \qquad \sum_i (f_{ext})_i - M\frac{dv}{dt} = 0 \qquad \sum_i f_i = 0$$

inercijalna sila
D`Alambert-ova sila

2. Zakon akcije i reakcije (III Njutnov zakon)



3. Zakon pomeraja: suma razlika pomeraja duž zatvorene putanje je 0

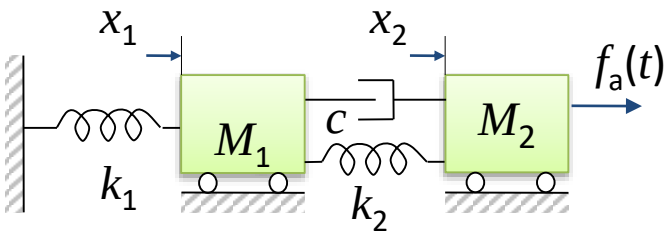


$$\sum_i (\Delta x)_i = 0$$

$$\underbrace{x_1}_{\substack{pomeraj \\ c_1}} + \underbrace{(x_2 - x_1)}_{\substack{pomeraj \\ c_2}} - \underbrace{x_2}_{\substack{pomeraj \\ k}} = 0$$

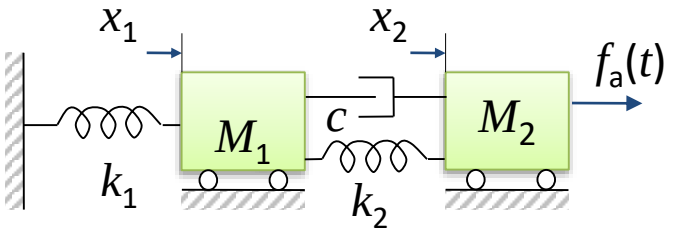
Primer 1

- Formirati model sistema

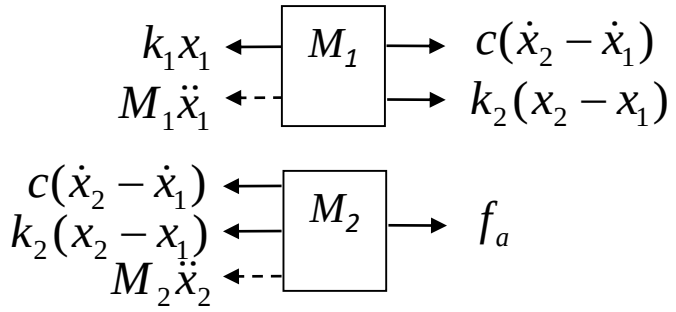


Dobijanje modela sistema – primer 1

- Kombinuju se zakonitosti elemenata i zakonitosti interakcije (međusobnih veza) elemenata



  - Za svako telo posmatramo sile koje na njega deluju

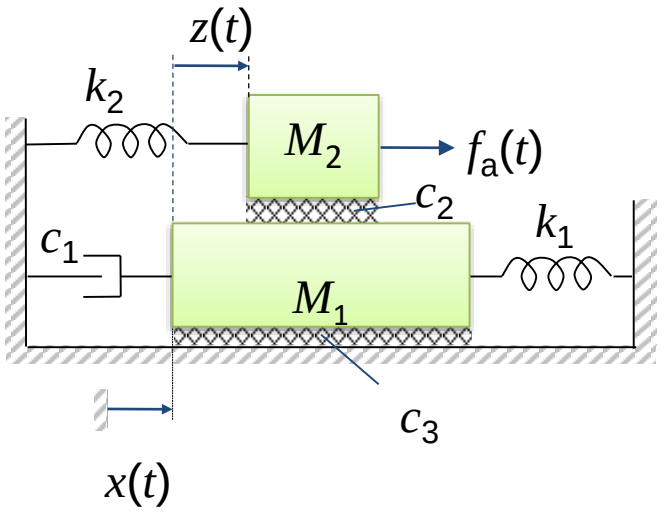


  - Na osnovu Dalamberovog zakona pišemo jednačine

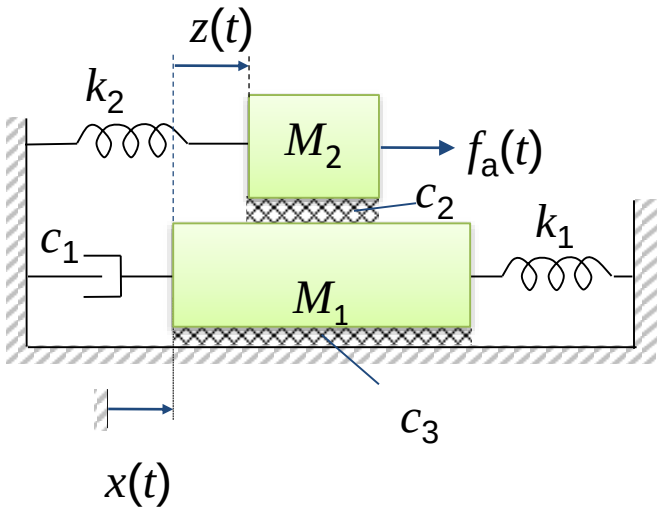
$$c(\dot{x}_2-\dot{x}_1)+k_2(x_2-x_1)-M_1\ddot{x}_1-k_1x_1=0$$
$$f_a(t)-c(\dot{x}_2-\dot{x}_1)-k_2(x_2-x_1)-M_2\ddot{x}_2=0$$

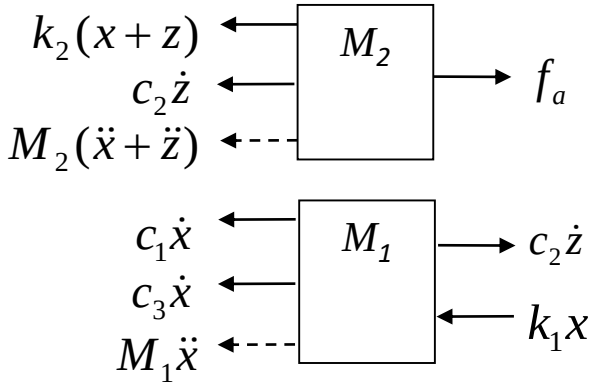
$$M_1\ddot{x}_1+c\dot{x}_1+(k_1-k_2)x_1-c\dot{x}_2-k_2x_2=0$$
$$-c\dot{x}_1-k_2x_2+M_2\ddot{x}_2+c\dot{x}_2+k_2x_2=f_a(t)$$

Primer 2

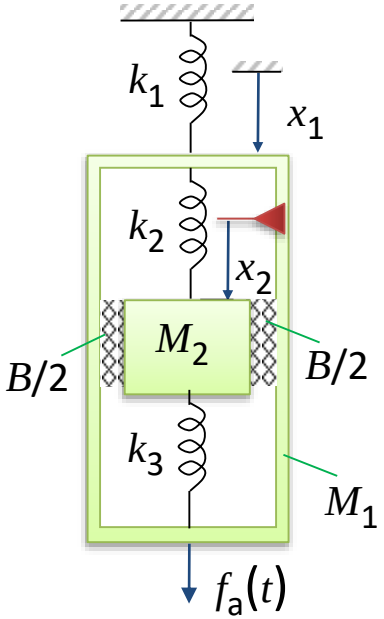


Primer 2

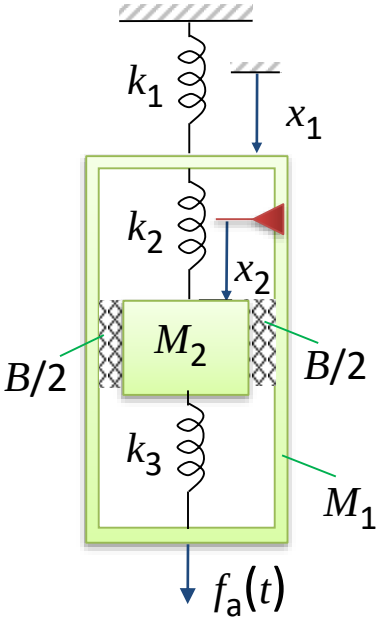

$z(t)$
k_2
M_2
$f_a(t)$
c_2
k_1
c_1
M_1
c_3
$x(t)$

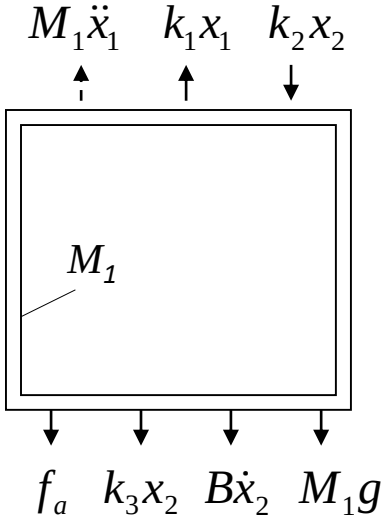

$k_2(x+z)$
$c_2\dot{z}$
$M_2(\ddot{x}+\ddot{z})$
M_2
f_a
$c_1\dot{x}$
$c_3\dot{x}$
$M_1\ddot{x}$
M_1
$c_2\dot{z}$
k_1x

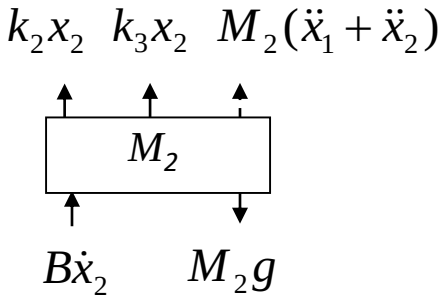
Primer 3



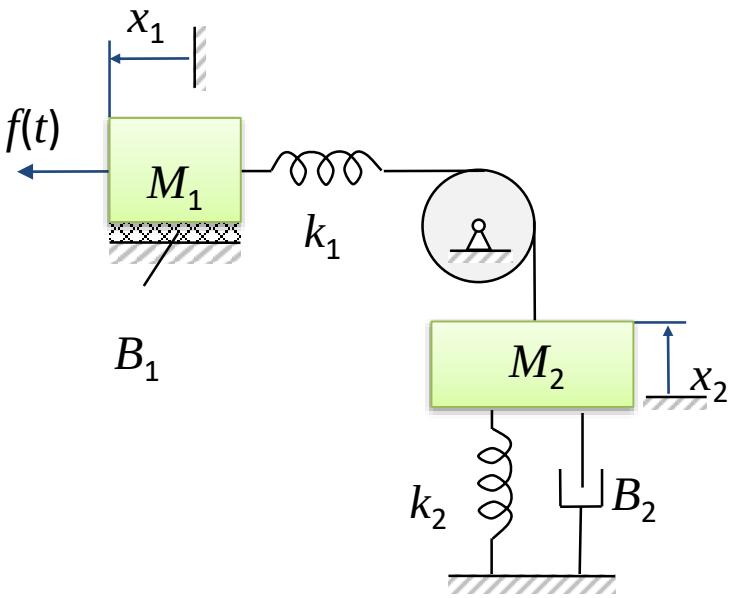
Primer 3


k_1
x_1
k_2
x_2
M_2
$B/2$
$B/2$
k_3
M_1
$f_a(t)$

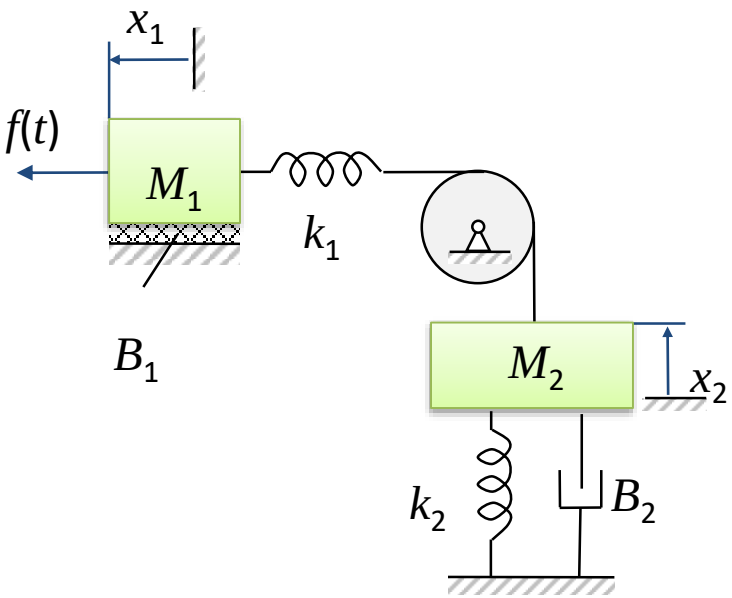

$M_1\ddot{x}_1$
k_1x_1
k_2x_2
M_1
f_a
k_3x_2
$B\dot{x}_2$
M_1g

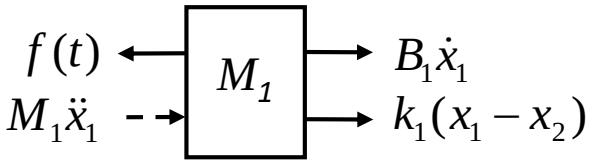

k_2x_2
k_3x_2
$M_2(\ddot{x}_1+\ddot{x}_2)$
M_2
$B\dot{x}_2$
M_2g

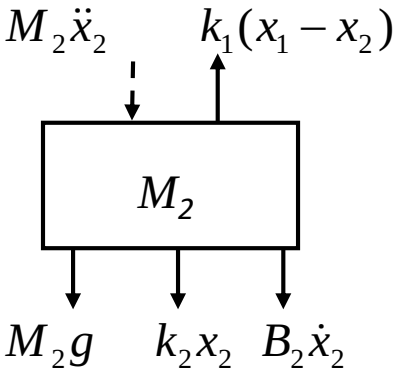
Primer 4



Primer 4


x_1
$f(t)$
M_1
k_1
B_1
M_2
x_2
k_2
B_2


$f(t)$
$M_1\ddot{x}_1$
M_1
$B_1\dot{x}_1$
$k_1(x_1 - x_2)$


$M_2\ddot{x}_2$
$k_1(x_1 - x_2)$
M_2
$M_2 g$
$k_2 x_2$
$B_2\dot{x}_2$