UNIVERZITET U NOVOM SADU
FAKULTET TEHNIČKIH NAUKA
KATEDRA ZA AUTOMATIKU I UPRAVLJANJE SISTEMIMA

Analiza ponašanja linearnih modela

Modeliranje i simulacija sistema

Upravljanje, modelovanje i simulacija sistema

Primer

- Data je funkcija prenosa G(s). Odrediti njen impulsni odziv.

$$G(s)=\frac{P(s)}{Q(s)}=\frac{7s^4+44s^3+108s^2+152.5s+83}{s^5+6.5s^4+20s^3+36.5s^2+34s+10}$$

$$Y(s)=G(s)U(s)$$

$$u(t)=\delta(t),\quad \ell\{u(t)\}=1$$

$$Y(s)=G(s)\cdot 1$$

$$y(t)=\ell^{-1}\{Y(s)\}=\ell^{-1}\{G(s)\}$$

```
>> P=[7 44 108 152.5 83];
>> Q=[1 6.5 20 36.5 34 10];

>> r=roots(Q)
r =
   -1.0000 + 2.0000i
   -1.0000 - 2.0000i
   -2.0000
   -2.0000
   -0.5000
```

$$\frac{P(s)}{Q(s)}=\frac{7s^4+44s^3+108s^2+152.5s+83}{(s+0.5)(s+2)^2(s^2+2s+5)}=$$

$$=\frac{a}{s+0.5}+\frac{b}{s+2}+\frac{c}{(s+2)^2}+2d\frac{s+1}{(s+1)^2+2^2}-2e\frac{2}{(s+1)^2+2^2}$$

Jednostruk realan pol

$$a = \lim_{s \to s_k} (s - s_k) \frac{P(s)}{Q(s)} = \lim_{s \to s_k} \frac{\frac{d}{ds}(s - s_k) P(s)}{\frac{d}{ds} Q(s)} = \left. \frac{P(s)}{\frac{d}{ds} Q(s)} \right|_{s = s_k}$$

```
>> a = polyval(P,-0.5) / polyval(polyder(Q),-0.5)
a =
     3
```

Višestruk realan pol

$$c = \lim_{s \to s_k} (s - s_k)^2 \frac{P(s)}{Q(s)}$$

```
>> Z = conv([1 0.5],[1 2 5]);   % Z=Q(s)/(s+2)^2
>> c = polyval(P,-2)/polyval(Z,-2)
c =
     4
```

$$K_{rm} = \frac{1}{(m-1)!} \left[\frac{d^{m-1}}{ds^{m-1}} (s - s_k)^p \frac{P(s)}{Q(s)} \right] \Bigg|_{s=s_k}$$

$$b = \frac{1}{(2-1)!} \left[\frac{d}{ds} (s+2)^2 \frac{P(s)}{Q(s)} \right] \Bigg|_{s=-2}$$

```
>> b = (polyval(conv(polyder(P),Z),-2)...
        -polyval(conv(polyder(Z),P),-2))/...
         polyval(conv(Z,Z),-2)
b =
     1
```

Konjugovano-kompleksni polovi

$$d + je = \left[\frac{P(s)}{\frac{d}{ds}Q(s)} \right] \Bigg|_{s=-\alpha+j\omega} \qquad \alpha = -1, \omega = 2$$

```
>> s=-1+2j; k1=polyval(P,s) / polyval(polyder(Q),s)
k1 =
   1.5000 - 0.2500i
>> d = real(k1);
>> e = imag(k1);
```

Odziv

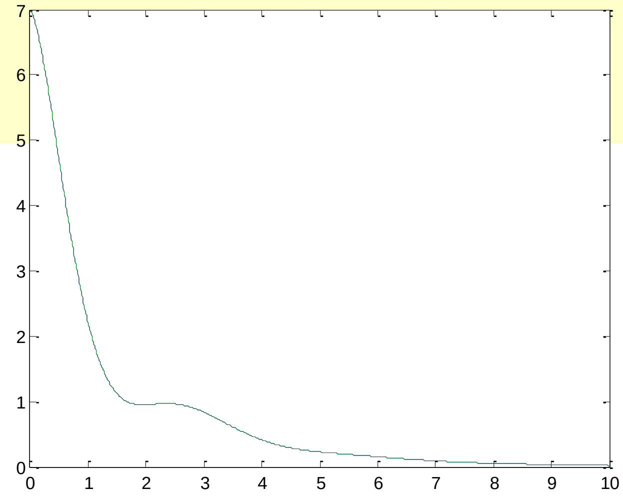
$$Y(s) = \frac{a}{s+0.5} + \frac{b}{s+2} + \frac{c}{(s+2)^2} + 2d\frac{s+1}{(s+1)^2+2^2} - 2e\frac{2}{(s+1)^2+2^2}$$

$$Y(s) = \frac{3}{s+0.5} + \frac{1}{s+2} + \frac{4}{(s+2)^2} + 3\frac{s+1}{(s+1)^2+2^2} + 0.5\frac{2}{(s+1)^2+2^2}$$

$$y(t) = 3e^{-0.5t} + e^{-2t} + 4te^{-2t} + 3e^{-t}\cos(2t) + 0.5e^{-t}\sin(2t)$$

```
>> t = 0:0.01:10;
>> y = a*exp(-0.5*t) + b*exp(-2*t) + c*t.*exp(-2*t) + ...
       2*d*exp(-t).*cos(2*t) - 2*e*exp(-t).*sin(2*t);

>> yi=impulse(P,Q,t);
>> plot(t,y,t,yi)
```



Upotreba funkcije residue

$$\frac{P(s)}{Q(s)} = \frac{r_1}{s-p_1} + \frac{r_2}{s-p_2} + \frac{r_3}{s-p_3} + \frac{r_4}{(s-p_3)^2} + \frac{r_5}{s-p_5}$$

$$\frac{P(s)}{Q(s)} = 2\beta\frac{s-\alpha}{(s-\alpha)^2+\omega^2} - 2\gamma\frac{\omega}{(s-\alpha)^2+\omega^2} + \frac{r_3}{s-p_3} + \frac{r_4}{(s-p_3)^2} + \frac{r_5}{s-p_5}$$

```
>> P=[7 44 108 152.5 83];
>> Q=[1 6.5 20 36.5 34 10];
>> [r,p,k]=residue(P,Q)
r =
   1.5000 - 0.2500i
   1.5000 + 0.2500i
   1.0000
   4.0000
   3.0000
p =
  -1.0000 + 2.0000i
  -1.0000 - 2.0000i
  -2.0000
  -2.0000
  -0.5000
k =
     []
```

$$r = \beta + j\gamma$$

$$p = \alpha + j\omega$$

$$Y(s) = 2\cdot 1.5\frac{s+1}{(s+1)^2+2^2} + 2\cdot 0.25\frac{2}{(s+1)^2+2^2} + \frac{1}{s+2} + \frac{4}{(s+2)^2} + \frac{3}{s+0.5}$$

Granične vrednosti

- Vrednost u t=0

$$y(0) = \lim_{s \to \infty} sY(s) = \lim_{s \to \infty} \frac{7s^5}{s^5} = 7$$

```
>> y(1)
ans =
      7
```

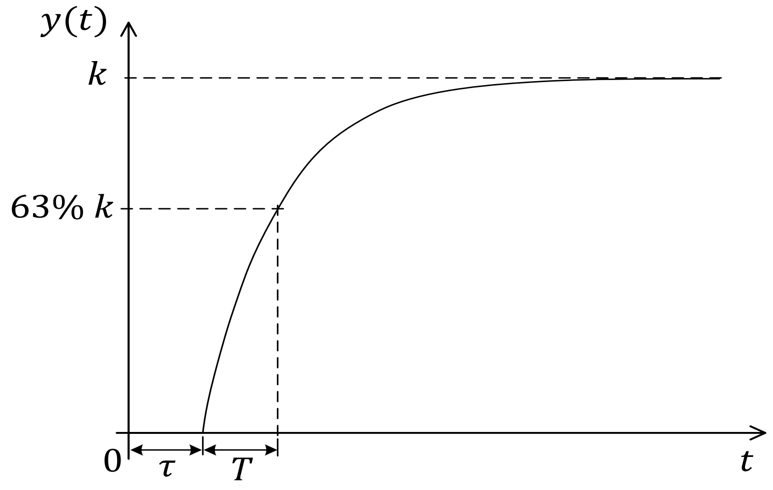
- Vrednost u t$\to \infty$

$$y(\infty) = \lim_{s \to 0} sY(s) = \frac{0}{10} = 0$$

```
>> dcgain([P 0],Q)
ans =
      0
```

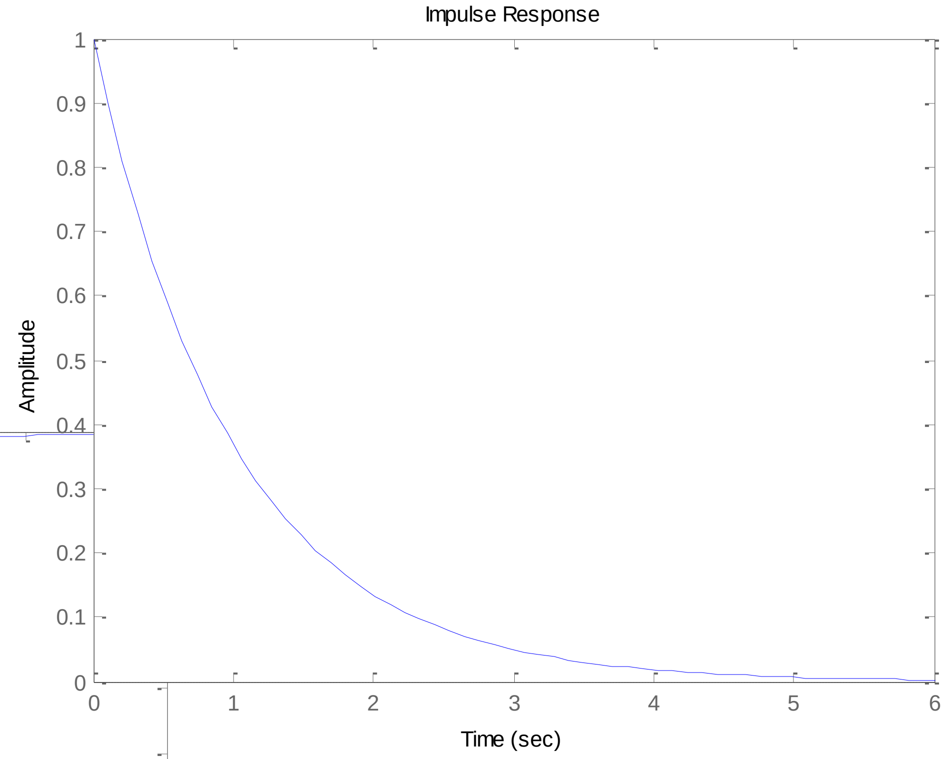
Model 1. reda

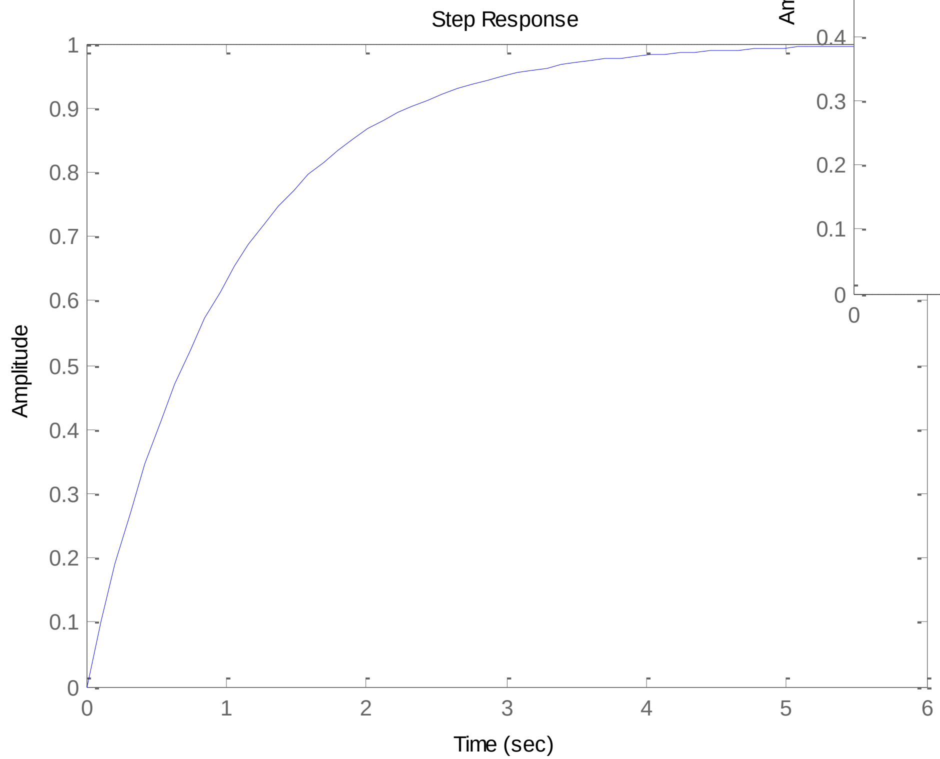
$$G(s) = \frac{k}{1+Ts}e^{-\tau s}$$



Sistem sa jednim realnim polom

$$G(s) = \frac{k}{1+Ts}e^{-\tau s}$$



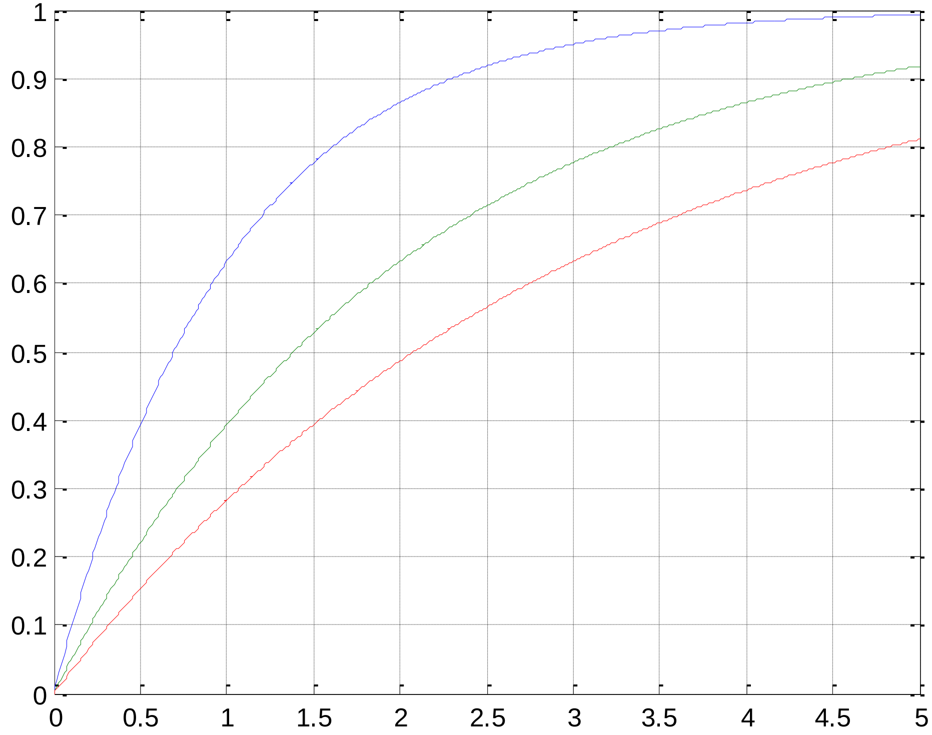


$$\tau = 0$$

```
>> P=1; Q=[1 1];
>> impulse(P,Q)
>> step(P,Q)
```

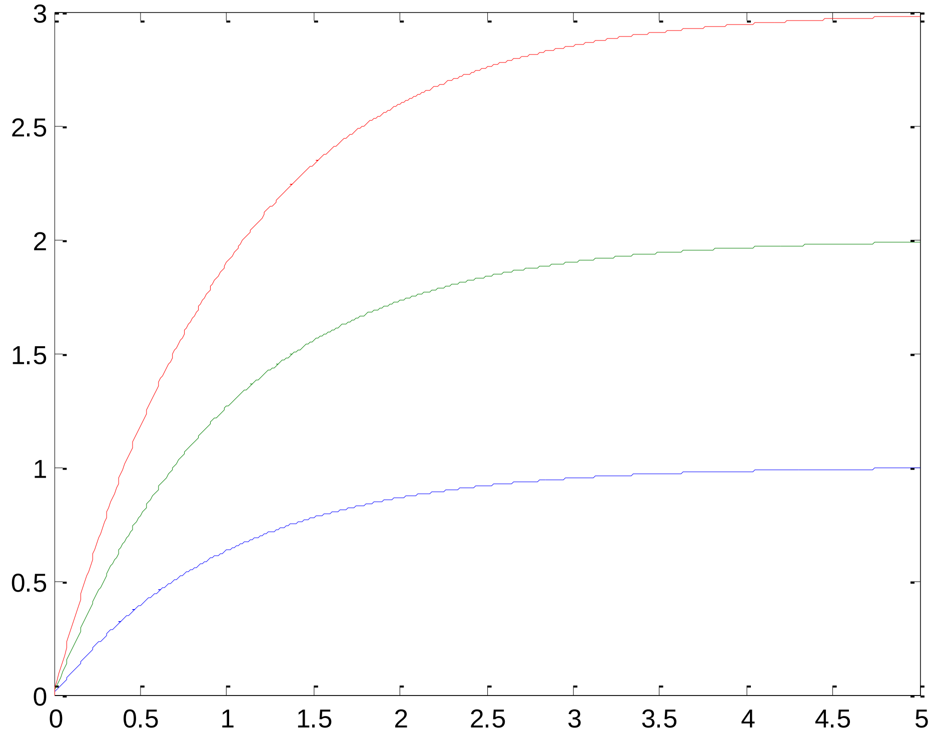
Sistem sa jednim realnim polom (2) promena vremenske konstante

```
t=0:0.01:5;
y1=step(P,[1*1 1],t);
y2=step(P,[1*2 1],t);
y3=step(P,[1*3 1],t);
plot(t,[y1 y2 y3])
grid
```



Sistem sa jednim realnim polom (3) promena pojačanja

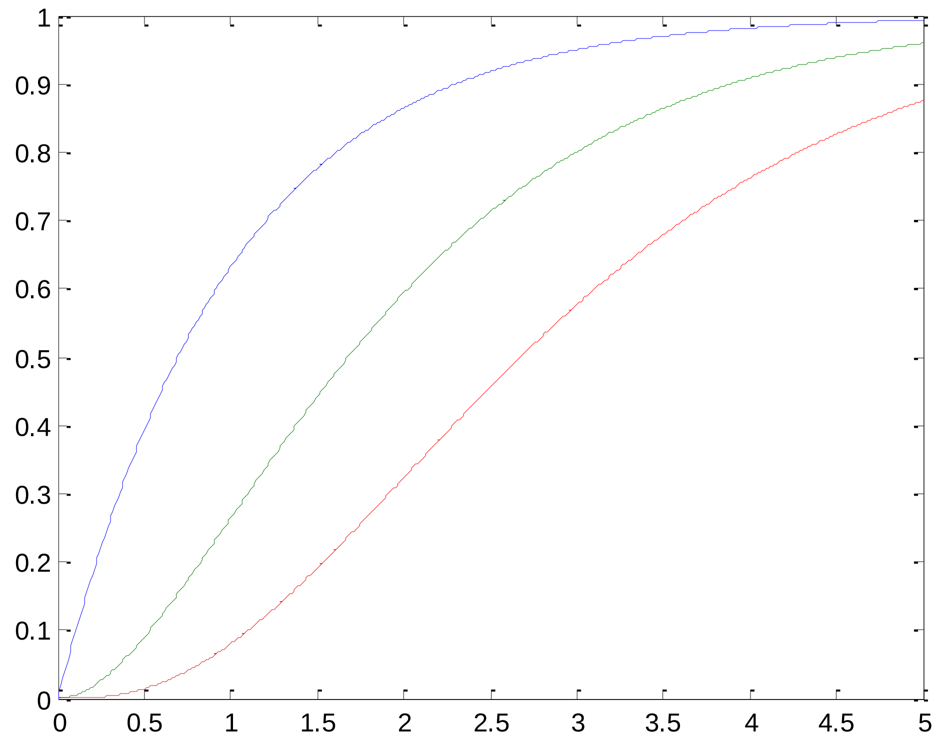
```
t=0:0.01:5;
y1=step(1,[1 1],t);
y2=step(2,[1 1],t);
y3=step(3,[1 1],t);
plot(t,[y1 y2 y3])
```



Sistem sa višestrukim realnim polom

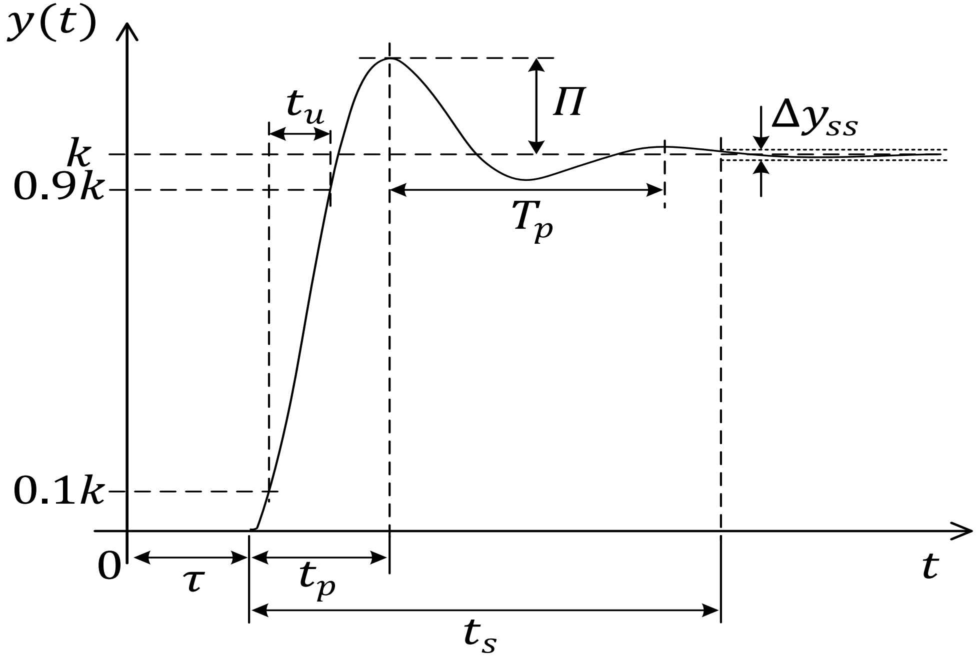
```
t=0:0.01:5;
y1=step(1,[1 1],t);
y2=step(1,[1 2 1],t);
y3=step(1,conv([1 1],[1 2 1]),t);
plot(t,[y1 y2 y3])
```

$$G(s) = \frac{k}{(sT+1)^r}$$



Model 2. reda

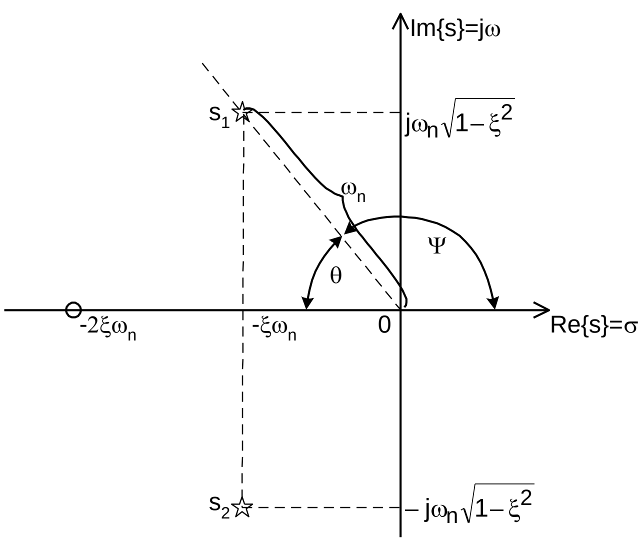
$$G(s) = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} e^{-\tau s}$$

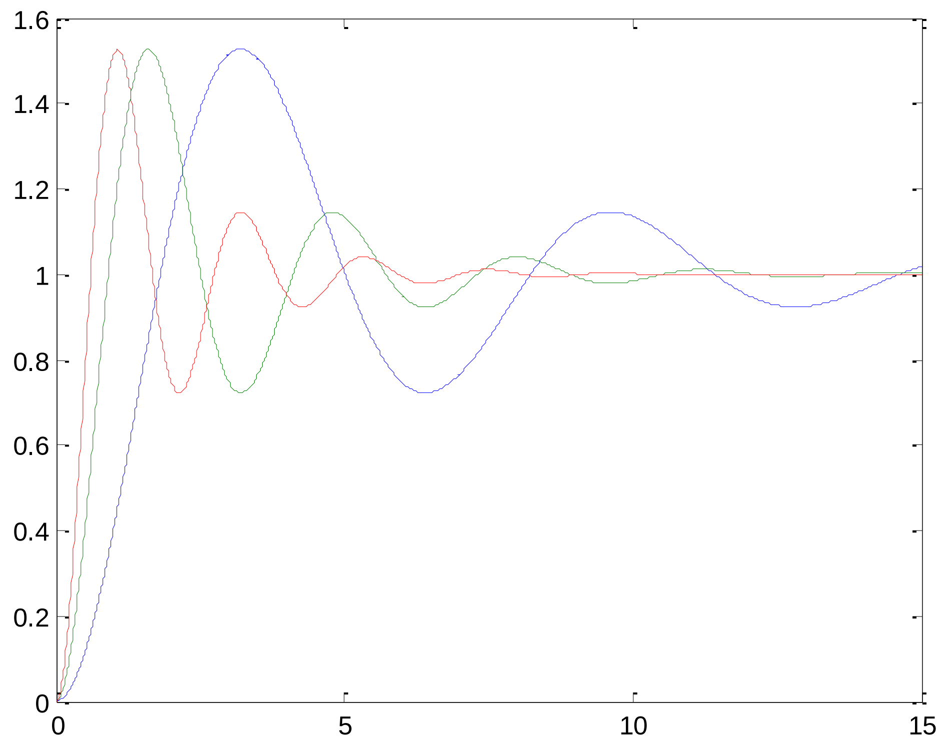


Par konjugovano-kompleksnih polova
uticaj prirodne učestanosti

```
t=0:0.01:15;
wn=1; ksi=0.2; y1=step(wn^2,[1 2*ksi*wn wn^2],t);
wn=2; ksi=0.2; y2=step(wn^2,[1 2*ksi*wn wn^2],t);
wn=3; ksi=0.2; y3=step(wn^2,[1 2*ksi*wn wn^2],t);
plot(t,[y1 y2 y3])
```

$$G(s) = \frac{\omega_n^2}{s^2 + 2s\zeta\omega_n + \omega_n^2}$$

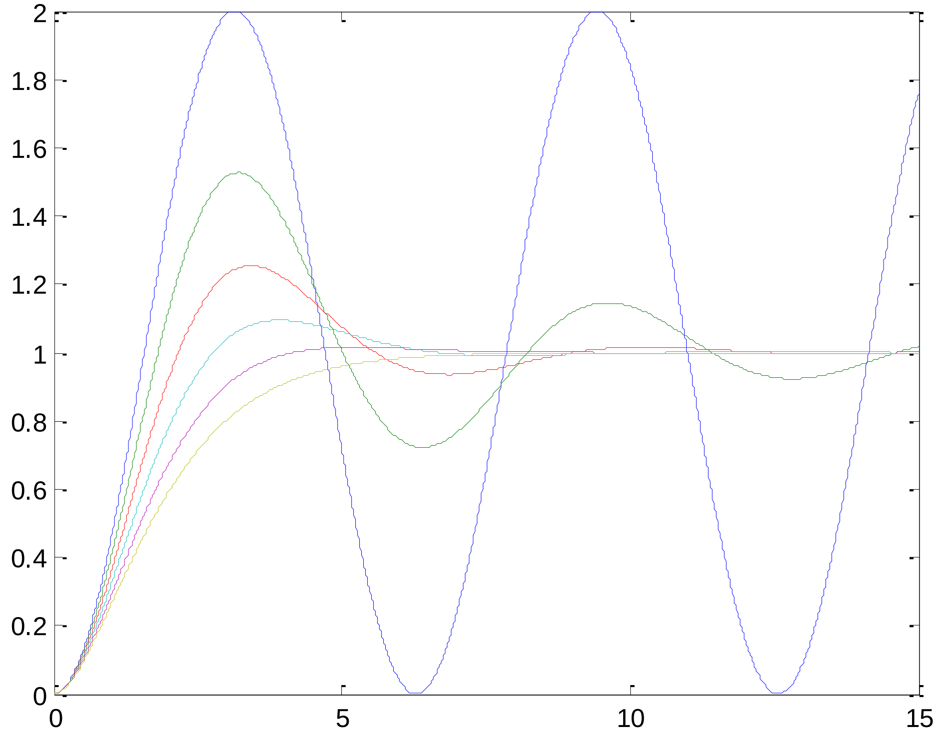




Par konjugovano-kompleksnih polova (2)
uticaj faktora relativnog prigušenja

```
wn=1; ksi=0.0; y1=step(wn^2,[1 2*ksi*wn wn^2],t);
wn=1; ksi=0.2; y2=step(wn^2,[1 2*ksi*wn wn^2],t);
wn=1; ksi=0.4; y3=step(wn^2,[1 2*ksi*wn wn^2],t);
wn=1; ksi=0.6; y4=step(wn^2,[1 2*ksi*wn wn^2],t);
wn=1; ksi=0.8; y5=step(wn^2,[1 2*ksi*wn wn^2],t);
wn=1; ksi=1.0; y6=step(wn^2,[1 2*ksi*wn wn^2],t);
plot(t,[y1 y2 y3 y4 y5 y6])
```

$$G(s) = \frac{\omega_n^2}{s^2 + 2s\zeta\omega_n + \omega_n^2}$$



Parametri odskočnog odziva

- Preskok Π % u odnosu na *vuus*
- Vreme kašnjenja T_k – signal dostigao 50% od *vuus*
- Vreme uspona T_u – od 10% do 90% *vuus*
- Vreme smirenja T_s – dostignuto *vuus* (±2-5%)
- Dominantna vremenska konstanta T_d
- Perioda oscilacija T

vuus - vrednost u ustaljenom stanju

$$T_d = \frac{1}{\zeta \omega_n}$$